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A Method of Measuring the Pressure Produced in the Detonation of High Explosives or by the Impact of Bullets

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X. *A Method of Measuring the Pressure Produced in the Detonation of High Explosives or by the Impact of Bullets.*

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The determination of the actual pressures produced by a blow such as that of a rifle bullet or by the detonation of high explosives is a problem of much scientific and practical interest but of considerable difficulty. It is easy to measure the transfer of momentum associated with the blow, which is equal to the average pressure developed, multiplied by the time during which it acts, but the separation of these two factors has not hitherto been effected. The direct determination of a force acting for a few hundred-thousandths of a second presents difficulties which may perhaps be called insuperable, but the measurement of the other factor, the duration of the blow, is more feasible. In the case of impacts such as those of spheres or rods moving at moderate velocities the time of contact can be determined electrically with considerable accuracy.* The present paper contains an account of a method of analysing experimentally more violent blows and of measuring their duration and the pressures developed.

If a rifle bullet be fired against the end of a cylindrical steel rod there is a definite pressure applied on the end of the rod at each instant of time during the period of impact and the pressure can be plotted as a function of the time. The pressure-time curve is a perfectly definite thing, though the ordinates are expressed in tons and the abscissæ in millionths of a second; the pressure starts when the nose of the bullet first strikes the end of the rod and it continues until the bullet has been completely set up or stopped by the impact. Subject to qualifications, which will be considered later, the result of applying this varying pressure to the end is to send along the rod a wave of pressure which, so long as the elasticity is perfect, travels without change of type. If the pressure in different sections of the rod be plotted at any instant (fig. 1) then at a later time the same curve shifted to the right by a distance proportional to the time will represent the then distribution of pressure. The velocity with which the wave travels in steel is approximately 17,000 feet per second. As the wave travels over any section of the rod, that section successively experiences pressures represented

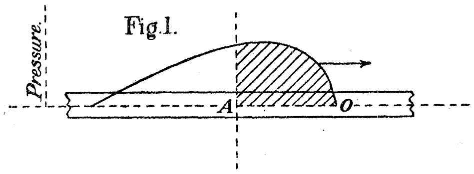

Fig. 1.

* Sears, 'Proc. Camb. Phil. Soc.,' vol. xiv. (1907), p. 257, and references there given.

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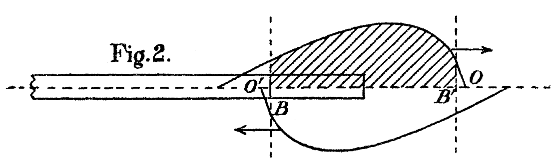
by the successive ordinates of the curve as they pass over it. Thus the curve also represents the relation between the pressure at any point of the rod and the time, the scale being such that one inch represents the time taken by the wave to travel that distance which is very nearly $\frac{1}{200,000}$ of a second. In particular the curve giving the distribution of pressure in the rod along its length is, assuming perfect elasticity, the same as the curve connecting the pressure applied at the end and the time, the scale of time being that just given.

The progress of the wave of stress along the rod is accompanied by corresponding strain and therefore by movement. It is easy to show that the same curve which represents the distribution of pressure at any moment also represents the distribution of velocity in the rod, the scale being such that one ton per square inch of pressure corresponds to about 1·3 feet per second of velocity. Until the wave reaches any section of the rod that section is at rest. It is then, as the wave passes over it, accelerated more or less rapidly to a maximum velocity, then retarded, and finally left at rest with some forward displacement. In this manner the momentum given to the rod by the application of pressure at its end is transferred by wave action along it, the whole of such momentum being at any instant concentrated in a length of the rod which corresponds, on the scale above stated (one inch = $\frac{1}{200,000}$ second), to the time taken to stop the bullet completely. Consider a portion of the rod to the right of any section A (fig. 1) which lies within the wave at the moment under consideration. The pressure has been acting on this portion since the wave first reached it, that is for a time represented by the length OA and equal to $\frac{\text{OA}}{\text{V}}$ where V is the velocity of propagation. The momentum which has been communicated to the part under consideration is equal to the time integral of the pressure which has acted across the section A, that is to the shaded area of the curve in the figure. The portion of the rod to the right of the section is continually gaining momentum at the expense of the portion to the left while the wave is passing, the rate of transfer at any instant being equal to the pressure.

When the wave reaches the free end of the rod it is reflected as a wave of tension which comes back with the same velocity as the pressure wave, and the state of stress in the rod subsequently is to be determined by adding the effects of the direct and of the reflected waves. Now suppose that the rod is divided at some section, B, near the free end (fig. 2), the opposed surfaces of the cut being in firm contact and carefully faced. The wave of pressure travels over the joint practically unchanged and pressure continues to act between the faces until the reflected tension wave arrives at the joint. The pressure is then reduced by the amount of the tension due to the reflected wave and as soon as this overbalances at



section B the pressure of the direct wave (which is the moment shown in the figure) the rod, being unable to withstand tension at the joint, parts there and the end flies off. The end piece has then acquired the quantity of momentum represented by the shaded area in the figure, equal to the time-integral of the pressure curve from O to B, less that of the tension wave during the time for which it has been acting, that is from O′ to B. The piece flies off with this amount of momentum trapped, so to speak, within it. If it be caught in a ballistic pendulum and its momentum thus measured we have the time integral of the pressure curve between the points B and B′ on the pressure-time curve which are such that they correspond to equal pressures on the rising and falling parts of the curve, while the time-interval between them is equal to that required for a wave to travel twice the length of the end piece. By taking end pieces of different lengths and measuring the momentum so trapped in each the area of the pressure-time curve over corresponding intervals can be obtained. In general the precise form of the curve itself cannot be deduced because the points of commencement of the several intervals are not known. Thus a given set of observations would be consistent with any one of the three forms shown in fig. 3 which can be derived from one another by shearing parallel to the base so that the intercept of any line such as AA′ is the same on all. But the maximum pressure and the total duration of the impact can always be obtained, and these are the most important elements. The maximum pressure is the limiting value of the average acting on a piece when the piece is very short, and the duration corresponds to twice that length of piece which just catches the whole of the momentum leaving the rod at rest. If the circumstances of the impact are such that the pressure is known to rise or to fall with great suddenness, the curve assumes the form I. or III. and its form may be determined completely from the observations.

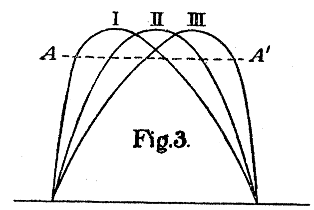


Fig.3.

This is the basis of the method described in the present paper. A cylindrical rod or shaft of steel is hung up horizontally by four equal threads so that it can swing in a vertical plane remaining parallel to itself. A short piece of rod of the same diameter is butted up against one end being held on by magnetic attraction but otherwise free. A rifle bullet is fired at, or gun-cotton is detonated near, the other end; the short piece flies off and is caught in a box suspended in a similar manner to the long rod. Suitable recording arrangements register the movement both of the long rod and of the box, and the momentum in each is calculated in the usual way as for a ballistic pendulum. Sufficient magnetic force to hold the end-piece in position is provided by putting a solenoid round the rod in the neighbourhood of the joint. The slight force required to separate the piece from the rod under these conditions may be neglected in comparison with the pressures and tensions set up, since these amount to several tons on the square inch, and, practically speaking, the joint will transmit the pressure wave unchanged but will sustain no tension.

Pressure Produced by the Impact of Lead Bullets.

The pressure which should be produced by the impact of a lead bullet can be predicted theoretically, and the study of this pressure was made rather with a view to checking the method than in the hope of discovering any new facts. At velocities exceeding 1000 feet per second lead behaves on impact against a hard surface practically as a perfect fluid.

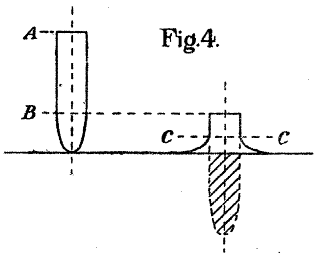


Fig. 4.

The course of the impact is shown in fig. 4. The base of the bullet at the moment of striking is at A; a little later it is at B. Assuming perfect fluidity the base of the bullet knows nothing of the impact at the nose and continues to move forward with unimpaired velocity. Hence the time elapsing between the two positions shown in the figure is $\frac{AB}{V}$. The momentum which has been destroyed up to this time is to a first approximation that of the portion of the bullet which has been flattened out, namely that portion shown shaded in the dotted figure. Knowing the distribution of mass along the length this is easily calculated. This simple theory is subject to some qualifications due partly to want of perfect fluidity, and partly to the fact that the sections of the bullet are not brought right up to the face and there stopped dead, as is assumed in the theory, but are more or less gradually retarded or deflected in the region of curved steam-lines at C. These corrections are, however, most conveniently introduced when comparing the theory with the experimental results.

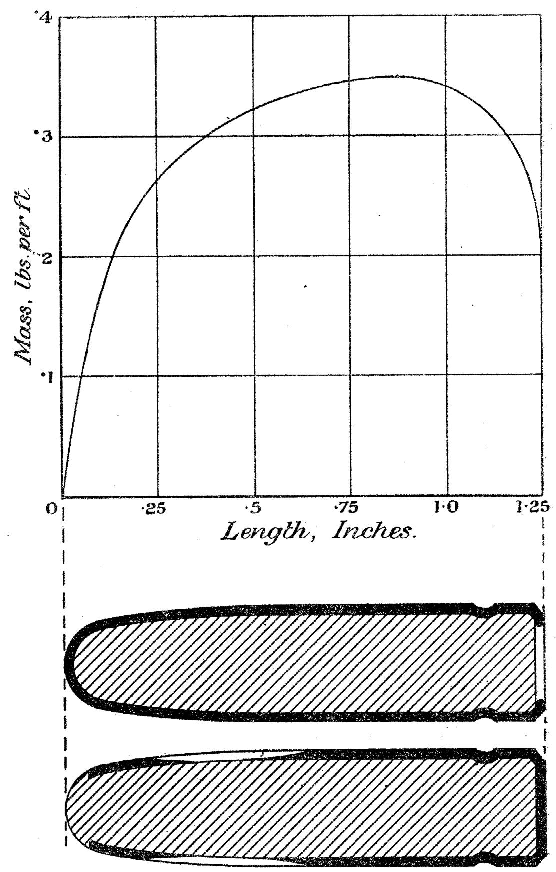


Fig. 5.

The bullets used were of two patterns, one the ordinary service form (Mark VI.) and the other a soft-nosed bullet supplied on the market for sporting purposes. Both are of lead, encased in nickel. Sections of the bullets are shown in fig. 5.* Sample bullets were sawn into sections, and the sections weighed. The distribution of weight along the length thus determined

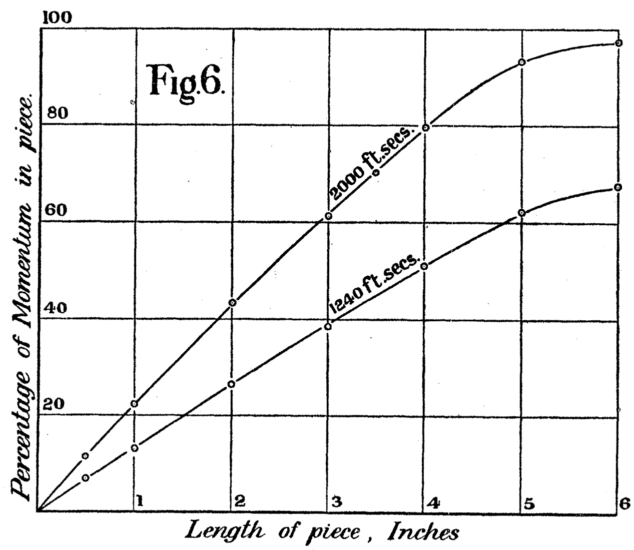
* The soft-nosed bullet (lower figure) has four longitudinal saw-cuts in the nickel casing; the section is taken through two of these cuts.

is shown in the curve fig. 5. The bullets were almost precisely alike both in regard to total weight (0·0306 lbs.) and distribution of weight along the length.

Most of the experiments were made with the service cartridge, in the service rifle, giving an average velocity of 2000 feet per second. These cartridges were very uniform, the range of variation in velocity being under one per cent. Some experiments were also made with cartridges giving velocities of about 1240 feet per second and 700 feet per second respectively.

The rod against which the bullet was fired was in most cases of steel containing C, 0·4 per cent.; Mn, 1·05 per cent. Its breaking strength was 37 tons per square inch with 24 per cent. elongation over 8 inches. The end of the rod was heated to a white heat in the forge and quenched and would then stand a large number of shots without serious damage. In some cases tool steel hardened, and tempered blue, was used, but it was found difficult to get the temper exactly right. The pieces butted to the end of the rod were usually of mild steel. For recording the movement of the rod and of the box in which the piece was caught each was fitted with a pencil which moved over a horizontal sheet of paper and the length of the mark was measured.

Assuming that the bullet strikes the rod fairly in the centre, and that the fragments are shot out radially, the total momentum recorded in rod and piece should be equal to the momentum of the bullet, which at 2000 feet per second is 61·2 lb. feet per second units. In fact, considerable variations were found in the total momentum. For instance, in 110 shots fired at a 1-inch rod, the maximum total recorded was 76,



the minimum 50, and the average 63. With a rod of $1\frac{1}{2}$ inches diameter, the variation was less; 61 shots showed a minimum of 59, a maximum of 70, and a mean of 62·5. High values are probably due to fragments being thrown back by irregularities in the surface of the rod, low values to slight errors in aiming. It was found, however, that with a piece of given length, the total momentum was shared between the piece and the rod in a nearly constant proportion, though the absolute values might vary widely. This is to be expected if the explanation just given of the irregularities is correct. For instance a cup-shaped cavity in the rod such as is formed after a large number of shots will give a high value for the momentum, but if not too pronounced it will not seriously affect the form of the relation between pressure and time.

The results have accordingly been reduced by taking in every case the percentages of the total momentum found in the piece. The following table gives details of one set of experiments. It was found that there was no systematic difference between the service bullets and the soft bullets, and the results for both types are included in the table :—

ROD, 1 inch diameter, 43 inches to 50 inches long. 2000 feet per second.

Length of piece.	Number of shots.	Percentage of total in piece.			Total momentum in rod and piece.		
		Maximum.	Minimum.	Mean.	Maximum.	Minimum.	Mean.
inches							
0·5	19	11·6	9·8	10·9	63	58	60
1·0	25	24·0	20·4	22·1	66	58	62
2·0	8	46·0	40·6	43·2	73	60	65
3·0	26	63·0	58·0	61·0	66	59	62
3·5	6	71·0	69·0	70·4	71	65	67
4·0	6	82·0	79·0	79·7	67	50	62
5·0	11	93·0	94·5	93·5	76	59	67
6·0	9	99	—	97·6	69	63	66

The mean percentages given in the third column of the table are plotted against length of piece in fig. 6. As the wave travels 2·04 inches in 10^{-5} seconds, 1 inch length of piece represents $0{\cdot}98 \times 10^{-5}$ seconds.* The slope of this curve represents pressure, and as already explained the maximum pressure is represented by the slope at the origin. This is 22 per inch, and assuming an average total momentum of 61·2 units the corresponding pressure is

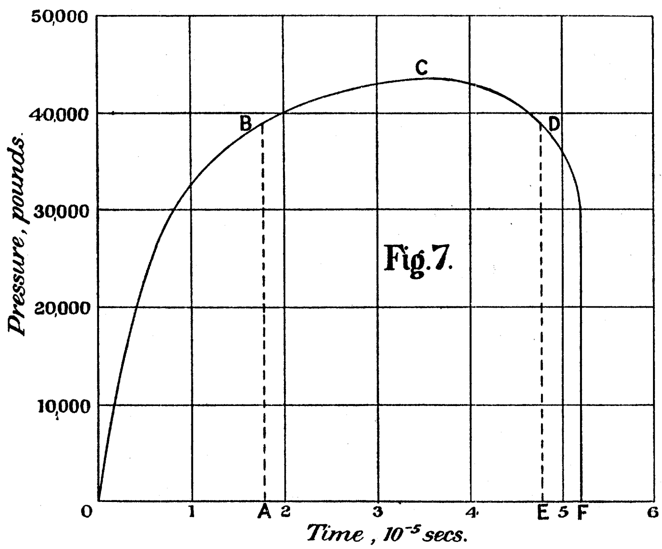
$$\frac{0{\cdot}22 \times 61{\cdot}2 \times 10^{5}}{32{\cdot}2 \times 0{\cdot}98} = 42{,}600 \text{ lbs. or } 19{\cdot}0 \text{ tons.}$$

It will also be noticed that the impact is practically complete in 6×10^{-5} seconds, $97\frac{1}{2}$ per cent. of the total being then accounted for in the piece.

According to the simple theory, which regards each element of the bullet as coming up to the end of the rod with its velocity v_0 unimpaired and there suffering instant

* The value of E for the mild steel of which the pieces were made was found to be $3{\cdot}00 \times 10^{7}$ lbs. per square inch. The density was 482 lbs. per cubic foot. Both determinations are probably right within 1 per cent. The velocity of propagation $\sqrt{\frac{E}{\rho}}$ is 17,000 feet per second.

stoppage, the pressure at any time is λv_0^2 where λ is the mass per unit length at the section which is undergoing stoppage at the time. The pressure-time curve, calculated in this way, is shown in fig. 7, in which the ordinates are proportional to the values of λ. This is the same curve as that giving the distribution of mass along the length of the bullet, the abscissa scale being such that the length OF within which the impact is



complete is equivalent to the time required by the bullet to travel its own length (1·25 inches) at a velocity of 2000 feet per second. This is $5{\cdot}2 \times 10^{-5}$ seconds. The maximum pressure corresponds to the maximum value of λ (0·35 lbs. per foot) and is

$$\frac{0{\cdot}35 \times 2000 \times 2000}{32{\cdot}2} = 43{,}500 \text{ lbs.}$$

which is $2\frac{1}{2}$ per cent. in excess of the value found by experiment. This difference is no more than can be accounted for by errors of observation.

The momenta which should according to theory be taken up by various lengths of piece are readily calculated from this curve. For instance, that corresponding to a 3-inch piece is the area ABCDE. The following table shows the results so obtained with the corresponding observed values. The momenta are reckoned as percentages of the total :—

Length of piece.	Percentage momentum in piece.	
	Calculated.	Observed.
inches		
3	65	61
4	84	80
5	98·5	93·5
6	100	97·5

The differences between the calculated and observed figures in this table are probably rather outside experimental errors. Especially is this the case as regards the 5-inch and 6-inch pieces. The impact seems to last appreciably longer than it ought.

The Effect of the Rigidity of the Bullet.

In the simple theory it is assumed that the bullet is absolutely fluid. In fact, it possesses a certain rigidity, partly because of the nickel casing and partly because of the viscosity of the lead the effects of which may be quite appreciable at such high speeds of deformation. The general effect of rigidity may be represented by saying that any section of the bullet requires to be subjected to an end-pressure P before it begins to deform at all, and this pressure must act across the section CC (fig. 4) where deformation is just beginning and where, if the bullet were really fluid, there would be no pressure. To a first approximation, P will be proportional to the area of the cross-section of the bullet which is undergoing deformation, that is to λ the mass per unit length in the plane CC. The pressure P is added to that due to the destruction of momentum, making a total pressure $P+\lambda v^2$ where λ is the mass per foot of the section of the bullet in the plane CC, v the velocity of that section. Further, the part of the bullet behind CC is being continually retarded by the pressure P, with the result that the hinder parts do not come up with unimpaired velocity v_0, as they would if the bullet were quite fluid, but with a diminishing velocity.

The general effect of this is obvious. In the early stages of the impact there has not been time for much retardation, and the pressure will be increased above the theoretical value by nearly the amount P. As the hinder parts come up, however, with less and less velocity, the fluid pressure term diminishes until the pressure falls below the theoretical value in spite of the rigidity term P. Applying this correction to a pressure curve such as that in fig. 7 in which the maximum pressure occurs somewhat late in the impact, it will be seen that the general effect will be to reduce that maximum, and also to make it flatter. Furthermore, since the tail of the bullet takes longer to reach the end of the rod, the impact will be prolonged beyond the theoretical time.

It is easy to get a rough idea of the magnitude of these effects. Assume that the bullet is cylindrical and of mass λ per unit length and that the deforming pressure is constant. Let x be the length of the bullet behind the plane CC (fig. 4). This portion is moving as a rigid body with acceleration $\ddot{x}$ and its equation of motion is

$$\lambda x\ddot{x} = -P,$$

which integrates in the form

$$\tfrac{1}{2}\dot{x}^2 = -\frac{P}{\lambda}\log x+\text{const.}$$

If l be the length of the bullet and $\imath_0$ its velocity on striking, and if we neglect the small distance between the plane CC and the end of the rod, the constant of integration is

$$\tfrac{1}{2}v_0^2+\frac{P}{\lambda}\log l,$$

and we have

$$1-\frac{\dot{x}^2}{v_0^2}=\frac{2P}{\lambda v_0^2}\log\frac{l}{x}.$$

From this $\dot{x}$ can be plotted in terms of x, and thence in terms of t. The total pressure $P+\lambda\dot{x}^2$ is then plotted in terms of the time.

As an example, take $\lambda = 0{\cdot}35$ lbs. per foot, $l = 1{\cdot}05$ inches which correspond to a bullet having the same mean density diameter and total mass as those used in the experiments. The pressure required to stop such a bullet at 2000 feet per second, if fluid, would be constant and equal to 43,500 lbs. If P be taken as $\frac{1}{20}$ of this, or 2170 lbs., and the curve plotted as described, it will be found that when $x = 0{\cdot}3l$ the hydrodynamical pressure λv^2 has dropped 12 per cent. making, after allowing the addition of 5 per cent. for the rigidity, a nett drop of 7 per cent. Furthermore, the momentum still left after a fluid bullet would have been completely set up is about 4 per cent. of the whole.

If corrections of this amount were applied to the calculated figures in the last section, the effect would be to make the observed maximum pressure about 4 per cent. too high, while the observed time of impact would be still slightly too long. It was found that to crush the cylindrical part of the service bullet in a testing machine required an end pressure of about 1800 lbs., but the nickel casing failed by buckling, whereas in the impact it apparently bursts and is torn into strips along the length of the bullet. The pressure required to deform the bullet in the latter case, after rupture is once started, is probably less than 2000 lbs. Thus, while the difference between the observed and calculated times of impact may undoubtedly be referred in part to rigidity, it is unlikely that the whole can be accounted for in this way.

Discussion of Errors Inherent in the Method of Experiment.

In calculating the pressure from the momentum in the piece which is thrown off the end of the rod it is assumed that the pressure wave transmitted along the rod represents exactly the sequence of pressures applied at the end, that it travels along the rod and through the joint without change of type, and that it is perfectly reflected at the other end. These assumptions are correct if the wave is long compared with the diameter of the rod, and if the pressure is uniformly distributed over the end, but are subject to certain qualifications in so far as these conditions are not fulfilled.

(*a*) *Effect of Length of the Rod.*—The mathematical theory of the longitudinal oscillations of a cylinder shows that a pressure wave of simple harmonic type is propagated without change, but the velocity of propagation depends on the wave-length. Because of the kinetic energy involved in the radial displacements, which is negligible when the wave is long compared with the diameter, the velocity diminishes with the wave-length. If the wave-length be $\frac{2\pi}{\gamma}$, and if the radius of the cylinder be a, the velocity is $\sqrt{\frac{E}{\rho}}(1-\frac{1}{4}\sigma^2\gamma^2a^2)$ correct to the square of γa.* In a wave of any form, the simple harmonic components move with different velocities, and the wave accordingly changes its form as it progresses.

Rough calculation of this effect on waves generally similar in form to that produced by the blow of the bullet, but of periodic character, showed that the change should not be very serious with rods of the lengths and diameters used in these experiments. It was, however, thought advisable to check this inference by direct experiment, and trials were therefore made with a rod 15 inches long and 1 inch diameter. The small mass of this rod precluded its use as a ballistic pendulum suspended in the ordinary way, it was therefore arranged to slide in bearings and to compress a spring buffer. Difficult questions arose as to the precise allowance which should be made for the kinetic energy given to the spring (which was of considerable mass) by the rod, and no attempt was therefore made to get an accurate measure of the total momentum. Instead of taking the fraction of this total which was trapped in the piece, the absolute values of the momenta so trapped were taken in a series of shots, in each of which, from the accuracy of the aiming and the absence of cupping in the end, it might be assumed that the total momentum was approximately equal to the average. The results are shown in the following table and are compared with the corresponding figures obtained with the long rod:—

ROD, 1 inch diameter. 2000 feet per second.

Length of piece.	Number of shots.	Momentum given to piece.			
		Short rod (15 inches).			Long rod.
		Mean.	Maximum.	Minimum.	Mean.
inches					
½	7	6·5	6·8	6·4	6·7
1	5	13·3	13·9	12·8	13·5
2	2	26·5	26·8	26·2	26·4
4	6	49·3	51·2	48·6	48·8
5	2	60·2	61·3	59·1	57·2

It is clear from these figures that there is no systematic difference between the results obtained with the two rods. The change, if any, between the forms of the wave when at 15 inches and at 45 inches from the end consists in a shearing of the

* LOVE, 'Mathematical Theory of Elasticity,' 2nd edition, p. 277.

whole curve as in the manner illustrated in fig. 3. Such a change of form—analogous to the change preparatory to breaking which a wave experiences as it advances into shallower water—would not be detected by these experiments, and it is not impossible that it occurs to some extent.

(*b*) *Reflection and Effect of the Joint.*—The simple harmonic pressure-wave which is propagated without change of type is accompanied by a distribution of shearing-stress across the section. This shearing-stress depends on the square of the ratio γa, and is small. That it plays no important part in these experiments is shown by the fact that if there be a joint in the long rod the results are unaltered. Such a joint transmits the pressure, but stops the shearing-stress part of the wave. As might be expected, it was found that the faces of the joint must be a carefully scraped fit if the wave is to pass it unaltered.

The small magnitude of the shearing-stress is the foundation of the assumption that the wave is perfectly reflected at the free end. Strictly accurate reflection is not possible. A reflected wave which is exactly the same as the incident wave, except that the signs of all the stresses are reversed, will when combined with the incident wave give no normal force over the free end. The shearing-stresses corresponding to the two waves do not, however, neutralise each other, but are added, hence accurate reflection can only be brought about by the application of a distribution of shear over the free end. The shear required is, however, of the order $\gamma^2 a^2$ and the experiment with the joint shows that its effects may be neglected.

(*c*) *Effect of the Diameter of the Rod.*—The pressure exerted by the bullet is confined to a comparatively small area in the centre of the end; whereas the pressure-wave travelling without change of type implies a nearly uniform distribution of pressure over the section. The question of the nature of the wave developed under such conditions seemed quite intractable mathematically, but from general considerations it appeared probable that it would not differ greatly from that of the wave originated by a uniform pressure distribution. In order to test this point

2000 feet per second.

Length of piece.	Percentage of momentum in piece.		
	$\frac{3}{4}$ inch.	1 inch.	$1\frac{1}{2}$ inch.
inches			
0·5	10·8	10·9	10·35
1·0	21·1	22·1	22·0
2·0	—	42·2	40·5
3·0	61·3	61·2	60·2
4·0	79·5	79·7	78
5·0	92·5	93·5	88
6·0	—	97·5	89

comparative tests were made with rods of $\frac{3}{4}$ inch, 1 inch, and $1\frac{1}{2}$ inch diameter. The lengths of the rods were roughly 48 inches, 43 inches, and 30 inches, respectively. The results are exhibited in the table on p. 447, in which the figures for the 1-inch rod are the same as those already given.

It will be seen that the diameter of the rod has no appreciable effect up to a length of 4 inches, but that for greater lengths the large rod gives appreciably lower values. In other words the apparent maximum pressure is not much affected by the diameter, and is presumably correctly given by all three rods, while the duration of the blow is largely overestimated by the $1\frac{1}{2}$-inch rod, and presumably somewhat overestimated by the other two, though as they are in substantial agreement on this point the error cannot be very large. It may be surmised that some at any rate of the difference between the observed and calculated times of impact is due to this cause, though, as already pointed out, the rigidity of the bullet is competent to account for part of it.

Experiments at Lower Velocities.

Measurements were also made with cartridges giving velocities of about 1240 feet per second and 700 feet per second respectively, the same types of bullet being used. The results for the 1240 feet per second cartridges are exhibited in the following table, which corresponds to that already given on p. 442 for the 2000 feet per second cartridges :—

Rod, 1 inch diameter, about 40 inches long. Velocity of bullets 1240 feet per second. (Mean of 5 shots : maximum 1257, minimum 1229.)

Length of piece.	Number of shots.	Percentage of total in piece.			Total momentum in rod and piece.		
		Maximum.	Minimum.	Mean.	Maximum.	Minimum.	Mean.
inches							
0·5	1	—	—	6·5	—	—	37·9
1	8	12·9	12·3	12·7	38·5	36·8	37·7
2	7	26·7	25·8	26·5	35·9	31·5	34·0
3	4	38·4	37·5	38·1	39·4	37·7	38·4
4	5	51·6	50·6	51·1	39·1	38·3	38·6
5	3	63·0	61·5	62·1	40·4	39·1	39·7
6	4	67·7	67·5	67·6	37·2	35·9	36·6
9	5	89	81·5	85·8	39·0	35·8	37·0

The mean total momentum registered (37 shots) is 37·7 units ; the calculated total is $1240 \times 0{\cdot}0306 = 38$ units.

The percentage figures are plotted in fig. 6 (curve marked "1240 feet per second").

The percentage of momentum trapped by short pieces is 13 per inch, and the corresponding maximum pressure for the normal velocity of 1240 feet per second is

$$\frac{0{\cdot}13 \times 38}{32{\cdot}2 \times 0{\cdot}98 \times 10^{-5}} = 15{,}700 \text{ lbs.}$$

The maximum pressure which should be exerted by a perfectly fluid bullet having the same mass and velocity is

$$\frac{0{\cdot}35 \times (1240)^2}{32{\cdot}2} = 16{,}700 \text{ lbs.}$$

The time taken by the bullet to travel its own length is $8{\cdot}4 \times 10^{-5}$ seconds. Thus if the bullet were perfectly fluid, the whole momentum should be trapped in a piece 9 inches long, whereas in fact only 86 per cent. is so trapped. The errors inherent in the method of experiment, which have been discussed in the last section, will all be less at the lower velocity. On the other hand the rigidity of the bullet will be relatively more important and probably suffices to account for much of the difference between the theoretical and observed times of impact.

The 700 feet per second bullets showed a maximum pressure of 5450 lbs., as compared with 5320 lbs. calculated. $54\frac{1}{2}$ per cent. of the momentum was trapped by a 9-inch piece. It was not possible to experiment with longer pieces, so that the time of impact in this case could not be determined.

It should be observed here that just after the piece has been shot off it tends to pull the rod after it by magnetic attraction, which of course still continues after the joint is broken, though it diminishes rapidly as the distance between piece and rod widens. The effect of this is to give more momentum to the rod and less to the piece than they would respectively possess as the effect of the blow alone. By measuring the amount of the magnetic pull when the piece is held at different distances from the rod, the current in the solenoid being the same as that used in the impact experiment, it is possible to estimate the amount of this effect. With 2000 feet per second bullets it is quite negligible, but when the velocities are lower particularly with long pieces, it necessitates a correction. This correction has been applied in the figures given above for the 1240 feet per second and 700 feet per second bullets.

Detonation of Gun-Cotton.

It is well-known that a charge of 1 lb. gun-cotton will shatter a mild steel plate 1 inch thick or more, if it be detonated in firm contact with it. The fracture is quite "short," like that of cast-iron, though the broken pieces are usually more or less deformed. Typical fractures of this kind obtained on plates of very good mild steel are illustrated in figs. 8, 9, 10, and 11. Figs. 8 and 9 are photographs of a plate $1\frac{1}{4}$ inches thick originally quite flat. It was broken by a slab of gun-cotton weighing 1 lb. which covered the section of the plate AB and was detonated in contact with that which became the convex face (lower face in fig. 9). Fig. 10 is a view of the broken edge of one of the two fragments. The plate shown in fig. 11 was a flat piece of boiler plate $1\frac{1}{4}$ inch thick. A slab of 1 lb. of gun-cotton was detonated against that which is the under side in the figure and the two pieces subsequently fitted together again and photographed. Thinner plates—*e.g.*, 1 inch thick—are usually

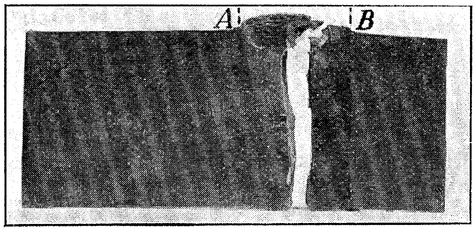


Fig. 8.

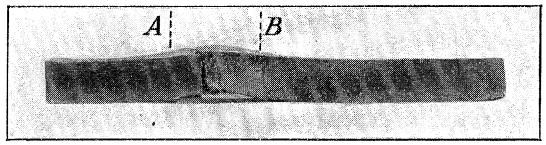


Fig. 9.

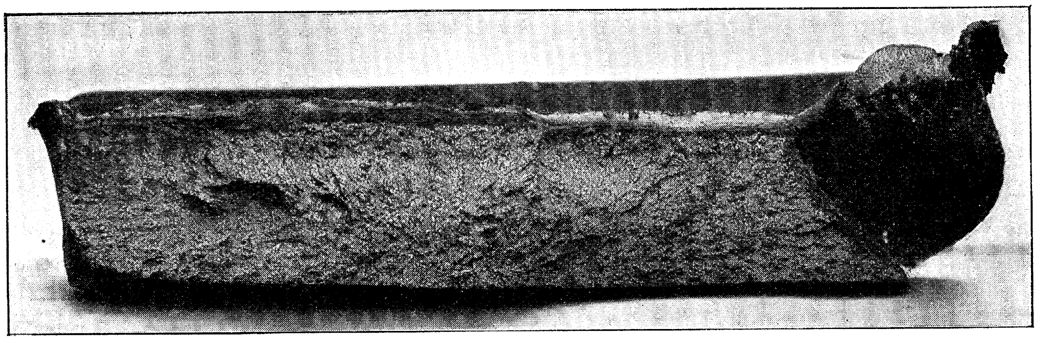


Fig. 10.

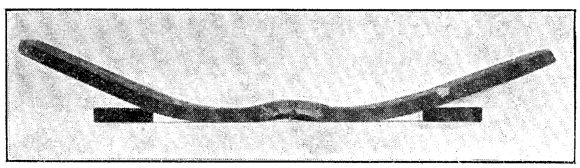


Fig. 11.

cracked in two places, one at each edge of the gun-cotton slab, and the portion covered by the slab is blown out of the plate, sometimes whole and sometimes shattered into pieces. The fact that no tamping is necessary suggests that the duration of the process of detonation is of the same order as the time taken by sound to travel an inch or less in air, so that during the conversion of the cotton into gas there is not time for much expansion.* If this be so, the maximum pressure

* The velocity of detonation of long trains of gun-cotton has often been measured and is variously estimated at 18,000 to 20,000 feet per second. If the same velocity obtained in the small primers they would be completely converted into gas in about 2×10^{-6} secs.

developed must be that which would be reached if the cotton were fired in a closed chamber of a volume not greatly exceeding that of the slab. The pressure is then dissipated with great rapidity by the expansion of the gas, which is resisted only by its own inertia and that of the surrounding air.

Experiments on the detonation of gun-cotton have been made by the method described in this paper. It has only been possible hitherto to use quite small charges and the results are a very rough approximation, but as they throw light on a matter

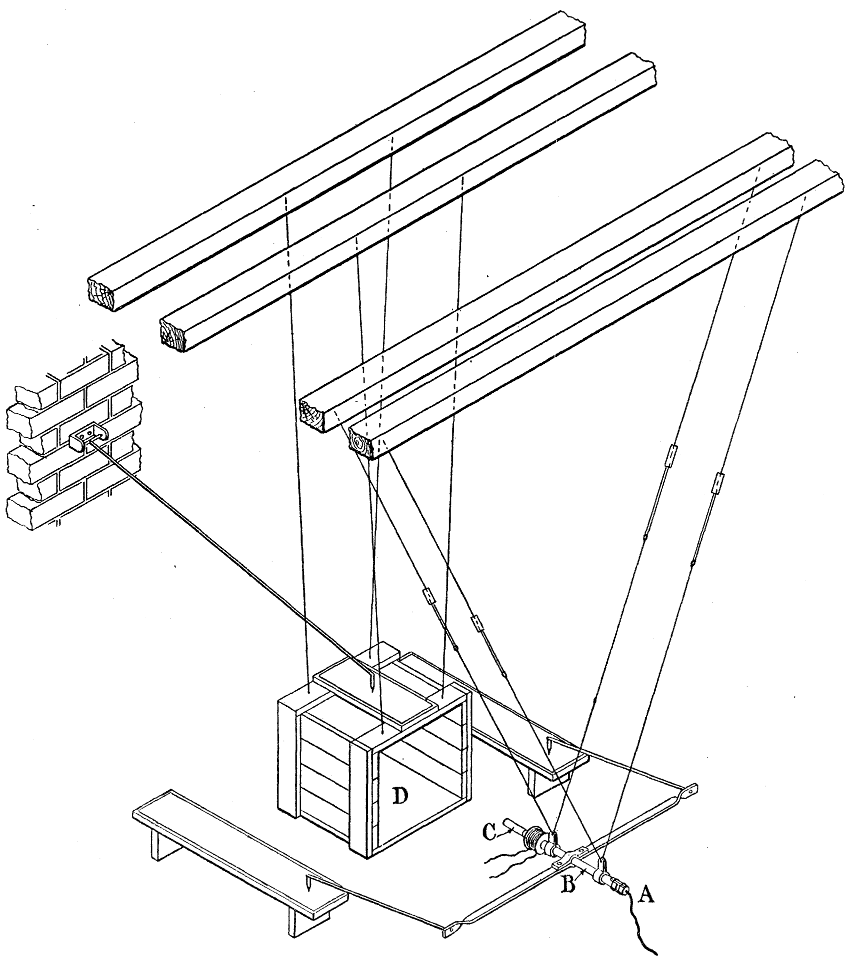


Fig. 12.

of which little is known I have thought it worth while to give them. Briefly, the conclusion is that the pressure at a point distant $\frac{3}{4}$ of an inch from the surface of one ounce of dry gun-cotton (a cylindrical "dry primer" about $1\frac{1}{4}$ inch diameter and $1\frac{1}{4}$ inches long), when detonated with fulminate, has fallen to less than $\frac{1}{5}$ of the maximum value within 2×10^{-5} seconds. At least, 80 per cent. of the blow has been delivered within that time. Over an interval of 10^{-5} seconds round about the time of maximum pressure the average pressure is about 30 tons per square inch, and the

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actual maximum is probably of the order of 40 tons per square inch. At a point on the surface the maximum pressure is at least twice as great, 80 tons per square inch.*

The arrangements are shown in fig. 12.

The gun-cotton cylinder A is fixed by short splints of wood opposite the end of the shaft B, which is of mild steel $1\frac{1}{4}$ inches diameter and from 15 to 30 inches long. This shaft is suspended as a ballistic pendulum with a pencil and paper for recording its

Length of piece.	Total nett momentum shaft and piece.	Percentage of total in piece.	Average percentage in piece.
inches			
3·85	40·7	86·6	90
	46·2	93	
	38·8	93	
	50·0	92	
	41·4	90	
	†31·2	90	
	†45·6	88	
3	60·0	89	89
	57·3	88	
	50·9	89	
	74·3	90	
2	36·7	81	83
	38·7	87	
	42·9	84	
	42·0	79	
0·95	40·8	57	57
	42·3	57	
	44·1	55	
	55·4	60	
	†44·6	57	
	†50·8	58	

movement. The end piece C, from $\frac{1}{2}$ to 6 inches long, is held on by magnetic attraction. The faces of the joint are a scraped fit. In line with the shaft is the box D, which is also suspended as a pendulum and provided with a recording pencil. Some part of the momentum given to the box is due to the blast from the gun-cotton; this

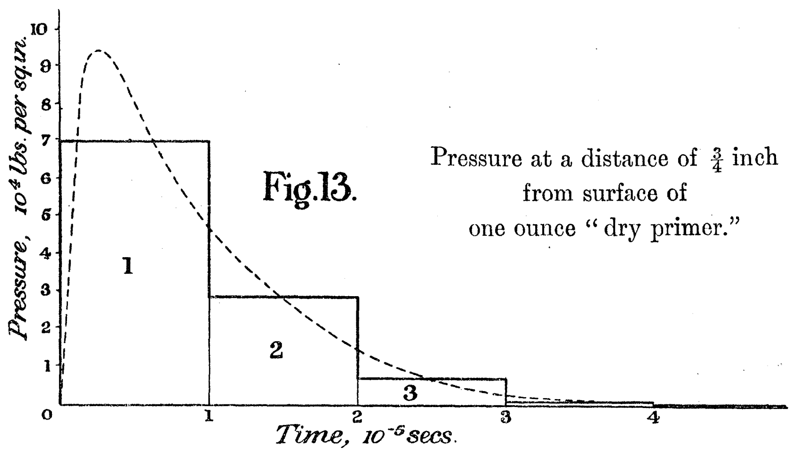
* The pressure developed by the explosion of gun-cotton in a vessel which it completely fills does not appear to have been measured. From measurements made with changes of lower density Sir ANDREW NOBLE estimates that it would be about 120 tons per square inch ('Artillery and Explosives,' p. 345). Allowing for the partial expansion during the process of detonation, this agrees fairly well with the pressure here determined.

† In these cases the air space between the gun-cotton and the end of the shaft was 1 inch. In all the others it was $\frac{3}{4}$ inch.

was estimated from experiments in which there was no piece on the end of the shaft. Separate experiments were also made to determine the effect of the blast on the supports of the shaft. The momentum accounted for by the blast is in each case deducted from the total recorded momentum to get the nett momentum due to the blow on the end of the shaft. This correction in the case of the box amounted to about 8·3 units with a 15-inch shaft, and 1·2 units with a 30-inch shaft. The correction for the blast on the supports of the shaft was 5 units.

The table on p. 452 gives the results of all the trials made with the gun-cotton about $\frac{3}{4}$ inch from the end of the shaft.

The total impulse of the blow when the air space is $\frac{3}{4}$ inch varies from about 35 to 70 units, the average being about 46 units. The percentages absorbed by the different end-pieces are, however, more nearly constant, and from them a rough approximation to the pressure wave transmitted by the rod in an average case may be constructed. As already explained the precise form of this curve depends on the way in which the pressure rises, but it may be assumed in this case that the pressure reaches its maximum in a time that is short even in comparison with the duration of the blow. Assuming an average total momentum of 45 units, fig. 13 has been constructed. The



area of the parallelogram marked 1 represents the momentum given to a 1-inch piece, the width of this parallelogram is 10^{-5} seconds and the height is the average pressure acting during the first 10^{-5} seconds. The parallelogram marked 2 represents the excess of the momentum given to the 2-inch piece over that given to the 1-inch piece and its height is the average pressure acting during the second 10^{-5} seconds. The dotted curve gives the same average pressures over the successive intervals of time. It is obviously largely conjectural, but it gives a rough idea both of the maximum pressure and of the duration of the blow.

The chief difficulty experienced hitherto in measuring by this method the pressures developed in the detonation of gun-cotton has been the permanent deformation of the end of the rod by the blow. No steel has yet been discovered which will stand, without flowing or cracking, the detonation of gun-cotton in contact with it, and even when a cushion of air $\frac{3}{4}$ inch thick is interposed some flow takes place.* In consequence of this, the pressure wave which emerges and is propagated elastically cannot be quite the same as the wave of applied pressure. It is easy to see that the general effect of the setting up of the end must be to deaden the blow, that is to reduce the maximum pressure and prolong its duration. In fig. 14, A is the (conjectural) curve representing

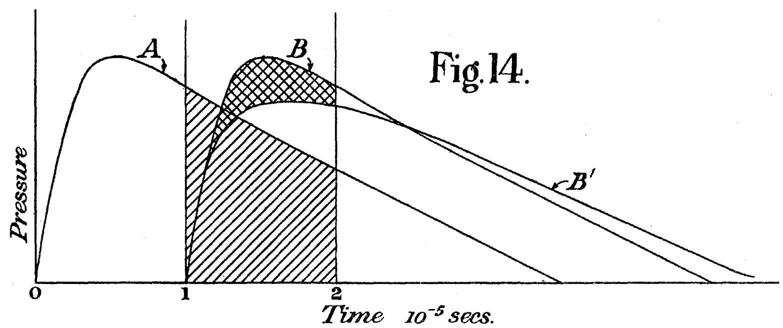


Fig. 14.

the pressure applied to the end of the rod. If the rod were perfectly elastic, the pressure across a section 2 inches from the end would be represented on the same time base by the curve B, which is the same as A, but moved 10^{-5} seconds to the right. The momentum in the end 2 inches at any time is the difference between the areas of the curves up to that time. For instance at 2×10^{-5} seconds it is represented by the shaded area under curve A. But if the end be not completely elastic, the higher pressures developed over section B will be less than those acting on the end at corresponding times. Thus the record of pressure over the section 2 inches from the end will be a curve such as B′ and the momentum in the end two inches at any time will be greater than it would be if the end were elastic by the difference between the areas of curves B and B′ which is double shaded in the figure. This extra momentum is transferred to the remainder of the rod later on, causing the curve B′ to rise above B. The curve B′ represents the wave of pressure actually sent along the rod. It is this curve which is determined by the method which has been described, and it is evident that that method under-estimates the maximum pressure and over-estimates the duration of the blow.

A few experiments were made with the gun-cotton touching the end of the shaft. The average total momentum given to the shaft and piece in this case is about 90 units or roughly twice as great as that transmitted through $\frac{3}{4}$-inch air-space. Of this total about 80 per cent. is caught in a piece 4 inches long, and about 50 per cent. in a piece 1 inch long. When the gun-cotton is at a distance of $\frac{3}{4}$-inch these figures are 90 and 60 respectively. The apparent duration of the pressure is therefore rather greater at the surface of the explosive. The setting up of the end of the shaft is, however, much more marked when the gun-cotton is in contact and it may

* This is when the steel is in the form of a shaft, so that there is no lateral support of the part subjected to pressure. It is, of course, possible to make a plate with hardened face which will withstand the attack of gun-cotton on a portion of the face.

be that the distribution of the pressure in time is not materially different in the two cases. If that were so, the maximum pressure developed on the surface of the gun-cotton would be 80 or 100 tons per square inch.

It is hoped that by the use of special steels it may be possible to give greater precision to these estimates of the amount and duration of the pressure produced by the detonation of gun-cotton in the open. Meanwhile the information already obtained as to the order of magnitude of these quantities is sufficient to throw some light on the nature of the fractures produced. The general result obtained may be expressed by saying that a gun-cotton cylinder $1\frac{1}{4}$ inches × $1\frac{1}{4}$ inches produces at its surface, when detonated, pressure of the average value of 100,000 lbs. per square inch lasting for $\frac{1}{50,000}$ second. Probably figures of the same sort of magnitude will describe the blow produced by the detonation of a slab $1\frac{1}{4}$ inches thick, one of whose faces is in contact with a steel plate. It may be that the pressure is greater and the duration correspondingly less, but this does not affect the point that the pressure is an impulsive one in its effect on the plate. That is, the effect of the pressure is to give velocity to the parts of the plate with which the gun-cotton is in contact but the pressure disappears before there has been time for much movement to take place. For instance, if the plate be 1 inch thick (mass 0·28 lbs. per square inch) a pressure of 100,000 lbs. per square inch acting on it for $\frac{1}{50,000}$ second will give a velocity of about 230 feet per second, and while the pressure is being applied it will move 0·028 inches.

The parts of the plate not covered by the gun-cotton are left behind and the strain set up by the forced relative displacement is the cause of the shattering of the plate. The magnitude of this strain, and of the consequent stress, depends (speaking generally), on the relation between the velocity impressed on the steel by the explosion and the velocity of propagation of waves of stress into the material. For instance, if the section AB (fig. 15) be given instantaneously a velocity of 200 feet per second and this velocity be maintained, the state of the plate after the lapse of $\frac{1}{100,000}$ second will be that represented diagrammatically by fig. 15. The section AB has moved forward relatively to the remainder by 0·002 feet. As soon as this section started moving a wave of shear stress started out from A into the parts of the plate to the left which had been left at rest by the blow. This wave travels in steel at 11,000 feet per second and will therefore in $\frac{1}{100,000}$ second get to C when AC = 0·11 feet. To the left of C the metal has not moved, the wave not having reached it; therefore the average shear in the section AC is $\frac{0·002}{0·11} = 0·018$. Under forces of this duration even mild steel has nearly perfect elasticity up to very

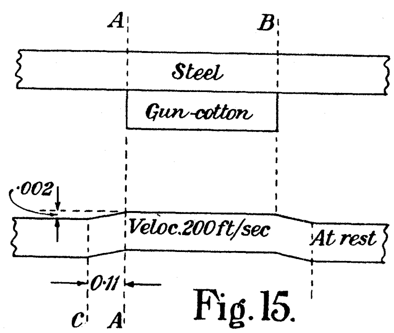


Fig. 15.

high stresses.* If it maintained its elasticity and continuity the shearing stress would be of the order $0{\cdot}018 \times 1{\cdot}2 \times 10^7$, or say 220,000 lbs. or 100 tons per square inch. This illustration is of course very far from representing the actual effect of suddenly giving velocity to a portion of a plate, the real distribution of stress would be far more complicated, but it gives an idea of the magnitude of the stresses which may be expected to arise. In static tests on mild steel, the material begins to flow as soon as the shearing stress exceeds about 10 tons per square inch and no stress materially greater than this can exist. But when the metal is forcibly deformed at a sufficiently high speed the shearing stress is increased by something analogous to viscosity and the tensile stress which accompanies it may be sufficient to break down the forces of cohesion and tear the molecules apart. Thus the steel is cracked, though in ordinary static tests it can stretch 20–30 per cent. without rupture, just as pitch, which can flow indefinitely if given time, is cracked by the blow of a hammer. The essence of the matter is the forcible straining of the substance at a velocity so high that it behaves as an elastic solid rather than as a fluid, thus experiencing stresses which are measured by the strain multiplied by the modulus of elasticity. The effect of gun-cotton on mild steel shows that in this material a rate of shear of the order 1000 radians per second is sufficient to cause cracking.

The most probable account of the smashing of a mild steel plate by gun-cotton is, then, that the plate is cracked before it has appreciably deformed, the cracks being caused by relative velocity given impulsively to different parts of the plate. Bending of the broken pieces occurs after the plate has cracked and the pieces have separated from one another. It is due to relative velocity in different portions of each piece which still persists after the initial fracture, and is taken up as a permanent set in each piece. In this connection the fracture shown in fig. 11 is instructive. It will be noticed that the general bend of the plate, after the pieces have been fitted together, is *opposite* to that which might at first sight be expected as the result of the blow in the middle. Inspection of such fractures leads to the conclusion just stated as to their history. The experiments on gun-cotton pressures described in this paper, though lacking in precision, supply I think the missing link in an explanation which is otherwise probable, namely, sufficient evidence that the blow may be regarded as an impulsive force communicating velocity instantaneously.

Most of the experimental work described in this paper was done by my assistant, Mr. H. Quinney. I also received valuable help in the earlier stages from Mr. A. D. Browne, of Queens' College, and from my brother Mr. R. C. Hopkinson, Trinity College. To these gentlemen I wish to express my obligation for aid without which it would hardly have been possible to carry out a research of this character. I have also to thank Sir Robert Hadfield, Mr. W. H. Ellis, and Major Strange for providing steel plates and shafts.

* Hopkinson, 'Roy. Soc. Proc.,' 74, p. 498.

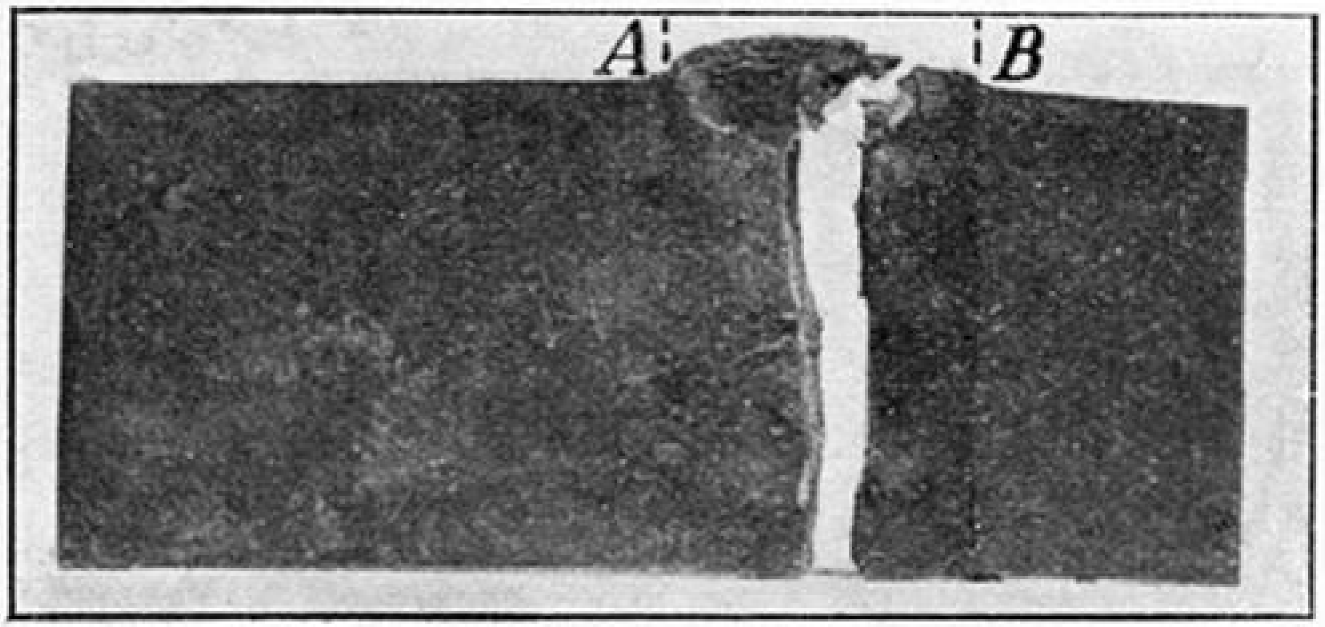


Fig. 8.

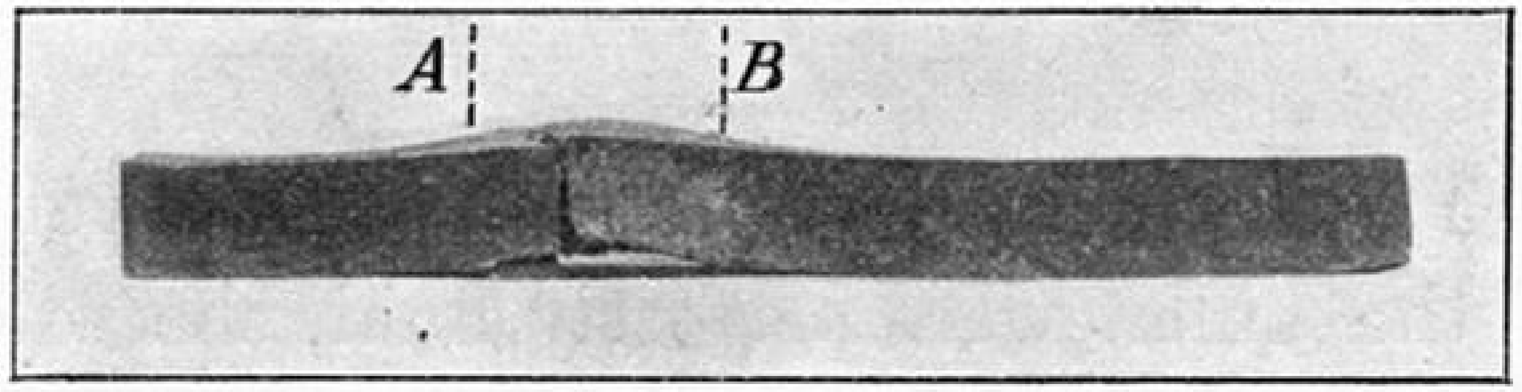
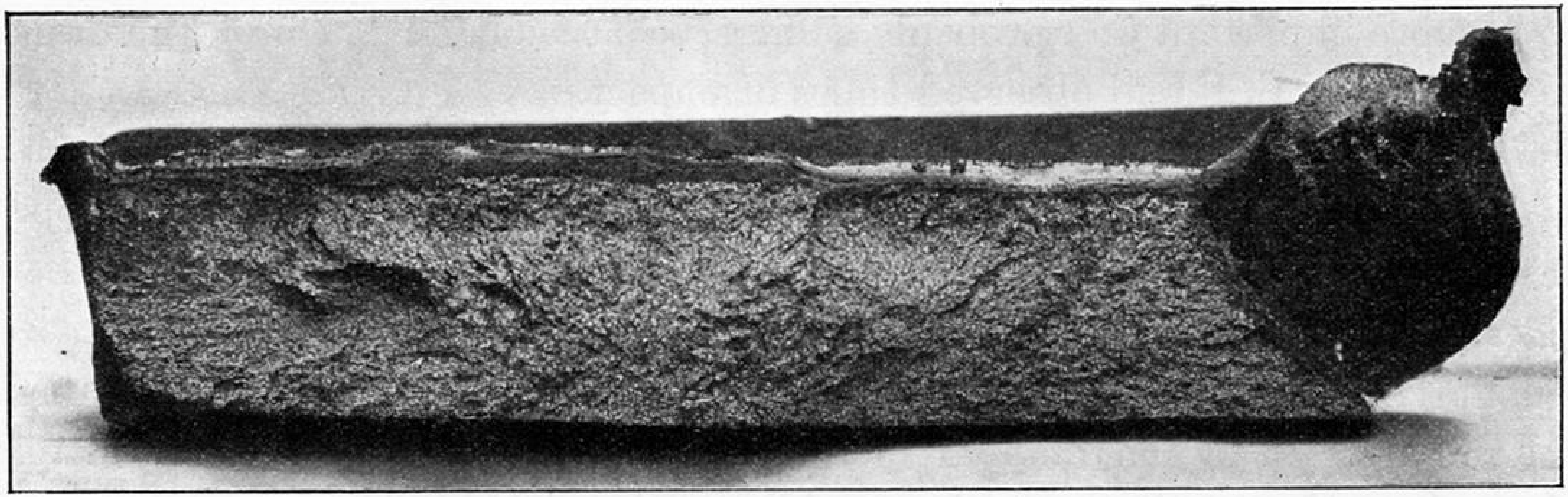


Fig. 9.



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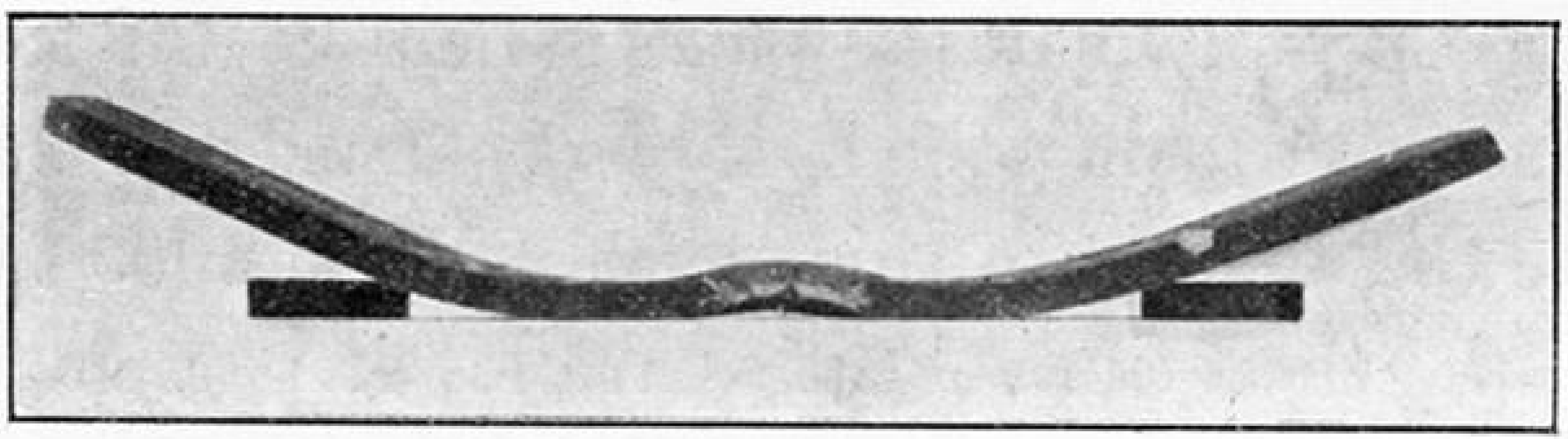

Fig. 11.